Date _____

In Class Activity

Probability

Group Member Names: _____

Part One – COIN FLIPS

1. In the past, you have probably flipped coins.
   a. If you have a fair coin, what is the probability of getting a heads on any one flip? _____

   b. What is the probability of getting a tails on any one flip? _____

   These types of probabilities are examples of ***theoretical probability***, which is calculated as the **number of ways a particular event occurs divided by the total number of possible outcomes**.

NOTATION:

Let ***E*** represent an **event** in a probability experiment.
Then, ***P(E)*** is read as the **probability that event E occurs**.
Note: The parentheses do *NOT* mean multiply. It is function notation like *f(x).*

2. Do you know anything about the largest and smallest possible values of probabilities?

   a. The probability of an event is never larger than what value? _____

   b. The probability of an event is never smaller than what value? _____

   c. Why do you think these are the boundaries of the possible values of probabilities?

3. Now, let's look at the notation for your answer for number 2. In the box below, it says that the probability of an event cannot be any larger than 1 (which is the same as 100% - a certain event) or any smaller than 0 (the same as 0% - an impossible event). Take a moment to understand the notation. Discuss it with your group as needed.

The probability of an event, P(E), must satisfy ***0 ≤ P(E) ≤ 1****.*
(*This says that the probability of any event must be between 0 and 1, inclusively.*)

$P(E) = 1$ means a ***certain event***, and $P(E) = 0$ means an ***impossible event***.

4.

a. Give an example of something that has a probability greater than 0.9. Use probability notation.

b. Give an example of something other than a coin flip that has a probability of 0.5.

c. Give an example of something that has a probability less than 0.05.

Part Two – DICE ROLLS

1. You have each been given a six-sided die.
   a. What is the *theoretical probability* of the die landing on any given number? _____
   b. Why?

2. Have each person in the group roll their die 10 times. Update the results into the table that follows.
   a. **Tally** what number came up each time, recording your **GROUP's** results.

   b. **Total** up all the tally marks for each number (1 through 6) for your **GROUP's** results.

   c. Calculate the **empirical probability** (or relative frequency) of each event for your **GROUP**. Write as a fraction and as a decimal rounded to 3 places.

NOTE: The ***empirical probability*** (or relative frequency) is defined as the **number of times an event occurs divided by the total number of incidents observed**.

   d. Calculate the **empirical probability** (or relative frequency) of each event for the **CLASS**. You can get this information from your instructor off the board. Write as a fraction and as a decimal rounded to 3 places.

   e. Write down the **theoretical probability** of each event, both as a fraction and as a decimal rounded to 3 places.

Put all these results in the table on the page that follows

Result on the face of the die →	**1**	**2**	**3**	**4**	**5**	**6**
a. Tally (keep track of the running count for all the **GROUP** member's results)						
b. Total Number of each value of the die (Frequency) for your **GROUP**						
c. Empirical Probability (Relative Frequency) for your **GROUP** *Write as a fraction and as a decimal rounded to 3 places.	$P(1)$ =	$P(2)$ =	$P(3)$ =	$P(4)$ =	$P(5)$ =	$P(6)$ =
d. Empirical Probability (Relative Frequency) for the **CLASS** *Write as a fraction and as a decimal rounded to 3 places.	$P(1)$ =	$P(2)$ =	$P(3)$ =	$P(4)$ =	$P(5)$ =	$P(6)$ =
e. Theoretical Probability *Write as a fraction and as a decimal rounded to 3 places.	$P(1)$ =	$P(2)$ =	$P(3)$ =	$P(4)$ =	$P(5)$ =	$P(6)$ =

3. Compare your group's empirical probabilities to the theoretical probabilities. Compare the class' empirical probabilities to the theoretical probabilities. Comment on what happens to empirical probabilities as the number of trials increases.

4. In a previous lesson (Helper or Hinderer), we used coin flips to model children making a decision in which they had no preference either way. Could we have used dice rolls instead? If so, how? If not, why not?

5.

TERMINOLOGY:

Success in a probability experiment is when the outcome that the researcher wants to happen, actually happens. Therefore, ***failure*** is when the desired outcome of a probability experiment does not happen.

Now, in the context of rolling a die, in your group, decide on which outcomes count as a success, and which count as a failure, assuming we are *modeling a decision with no preference* (either outcome, success or failure, is equally likely to occur). Record those here below:

a. **SUCCESS** means rolling the numbers: _____

b. **FAILURE** means rolling the numbers: _____

Use these outcomes to generate a table of successes and failures, including both **GROUP** and **CLASS** results.

	SUCCESS	**FAILURE**
Total Number for your **GROUP** (Frequency)		
Empirical Probability for your **GROUP** (Relative Frequency) Fraction and rounded to 3 places.		
Empirical Probability for the **CLASS** (Relative Frequency) Fraction and rounded to 3 places.		
Theoretical Probability		

6. Compare your group's and the class's ***empirical*** probabilities to the ***theoretical*** probability for this scenario. What is similar? What is different?

7. The box below states an important probability rule called the Law of Large Numbers.

> The **Law of Large Numbers** says:
> *As the number of trials increases, the empirical probability tends to get closer to the theoretical probability.*

Compare your group's empirical probabilities to the class' empirical probabilities in the context of the Law of Large Numbers. Did your results agree with the Law of Large Numbers? Summarize your conclusions here.

8. We've now seen two different ways to model a scenario in which the probability of success and the probability of failure are both 1/2. We used coin flips and dice rolls. Comment on a third way to model a fifty-fifty scenario.

9. Suppose you wanted to model a scenario where the probability of success is 1/4 and the probability of failure is 3/4. Suggest at least two ways to do so.

Part Three – TOTAL PROBABILITIES

1. Consider once again the event of flipping a fair coin.
   a. What is the **sum** of the ***theoretical*** probabilities of heads and tails for a coin flip?

      $P(\text{Heads}) + P(\text{Tails}) =$ _____ + _____ = _____

   b. What is the **sum** of the **GROUP's *empirical*** probabilities of getting 1, 2, 3, 4, 5, or 6 for the dice rolls? (Refer back to your table on page 23, from Part Two, number 2, row c)

      $P(1) + P(2) + P(3) + P(4) + P(5) + P(6) =$

      _____ + _____ +_____ +_____ +_____ +_____ = _____

c. What is the **sum** of the **CLASS's *empirical*** probabilities of getting 1, 2, 3, 4, 5, or 6 for the dice rolls? (Refer back to your table on page 23, from Part Two, number 2, row d)

$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) =$

_____ + _____ +_____ +_____ +_____ +_____ = _____

d. What is the **sum** of the ***theoretical*** probabilities of getting 1, 2, 3, 4, 5, or 6 for the dice rolls? (Refer back to your table on page 23, from Part Two, number 2, row e)

$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) =$

_____ + _____ +_____ +_____ +_____ +_____ = _____

2. You should have discovered that:

> ***The total probability of all the outcomes of a probability experiment must be 1.***

3. The box below summarizes the two probability rules discovered in this lesson. Make sure you can say them *in your own words* and understand how they are applied.

> ***Basic Probability Rules:***
> ***1.*** *The probability of an event, P(E), must satisfy $0 \le P(E) \le 1$.*
> ***2.*** *The total probability must be 1.*

Part Four – ADDING PROBABILITIES

1. Refer back to your group's table of outcomes for the dice rolls (from Part Two, number 2, page 23) What is your **GROUP's *empirical*** probability of getting an outcome that is *even or at least 5*? (Show your work – be clear with the details)

2. Use probability notation similar to what is written below to write out the addition rule for (empirical) probability when *A* = "even" and *B* = "at least 5." Fill in the appropriate numbers and show that this rule gives the same value as you got in the previous problem.

> The **Addition Rule (Inclusion/Exclusion) for Probability** says:
> $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
> or stated another way: $P(A \text{ or } B) = P(A) + P(B) - P(\text{both } A \text{ and } B)$
> *The probability that either event A or event B, or both, happens is the sum of the individual probabilities minus the probability that they both happen at the same time.*

3. Find the empirical probability of rolling either a 3 or a 4. Why don't we have to subtract anything when we add these probabilities together?

For extra practice:

4. Find the ***theoretical*** probability of getting either an even number or at least 5, when rolling a fair, six-sided die. (Show your work and use the appropriate probability notation.)

5. Find the theoretical probability of getting either a Queen or a Diamond when drawing one card from a standard 52-card deck of cards. (Show your work and use appropriate probability notation.)

MAIN IDEAS: List the Main Ideas for Today's Lesson